Math 146 C – Eigenvalue Problems

$y'' + \lambda y = 0$

$\lambda > 0 \Rightarrow \pm i\sqrt{\lambda} \Rightarrow y(x) = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x$

$\lambda = 0 \Rightarrow 0, 0 \Rightarrow y(x) = c_1 + c_2 x$

$\lambda < 0 \Rightarrow r = \pm\sqrt{-\lambda} \Rightarrow y(x) = c_1 \cosh\sqrt{s}x + c_2 \sinh\sqrt{s}x \quad s = -\lambda$

$\frac{(n-\frac{1}{2})\pi}{L}x$ for mixed conditions

IVP – time dependent

$y(t_0) = y_0 \quad y'(t_1) = y_1$

PDE: $\nabla^2 u = 0$ Laplace

$u_t - \nabla^2 u = 0$ Heat

$u_{tt} - \nabla^2 u = 0$ Wave

BVP – spatial dependent

$y(\alpha) = y_0 \quad y(\beta) = y_1$

$y(\alpha) = y_0 \quad y'(\beta) = y_1$

$y'(\alpha) = y_0 \quad y(\beta) = y_1$

$y'(\alpha) = y_0 \quad y'(\beta) = y_1$

BVP: $\phi''(x) + \lambda\phi = 0$

BC's	$\phi(0) = 0$, $\phi(L) = 0$	$\phi_x(0) = 0$, $\phi_x(L) = 0$	$\phi(0) = 0$, $\phi_x(L) = 0$	$\phi_x(0) = 0$, $\phi(L) = 0$
Eigenvalues	$\left(\frac{n\pi}{L}\right)^2$	$\left(\frac{n\pi}{L}\right)^2$	$\left(\frac{(n-\frac{1}{2})\pi}{L}\right)^2$	$\left(\frac{(n-\frac{1}{2})\pi}{L}\right)$
Eigenfunctions	$\sin\left(\frac{n\pi x}{L}\right)$	$\cos\left(\frac{n\pi x}{L}\right)$	$\sin\left(\frac{(n-\frac{1}{2})\pi x}{L}\right)$	$\cos\left(\frac{(n-\frac{1}{2})\pi x}{L}\right)$

Math 146C - Two Pt. BVP and Linear Systems of Algebraic Eqns.

10.1

Linear Systems: $A\vec{x} = \vec{b}$ (non-homog)

$\vec{b}$ vector $(n \times 1)$
A a matrix $(n \times n)$
$\vec{x}$ is the soln vector

① A non singular $(\det \neq 0)$ (inverse exists) $\Rightarrow$ unique soln unless $\vec{b}$ satisfies $(\vec{b}, \vec{y}) = 0$ — of ∞ many

② A singular $(\det = 0)$ (no inverse) $\Rightarrow$ no soln (

where $A^* \vec{y} = 0$ (adjoint) p 379

Homog: $A\vec{x} = \vec{0}$

Always has a soln (namely trivial soln)

③ A non singular $\Rightarrow$ only solution

④ A singular $\Rightarrow$ ∞ many, nontrivial solns

General

$$y'' + p(x)y' + q(x)y = g(x) \qquad y(\alpha) = y_0, \quad y(\beta) = y_1$$

Ex1) Eigenvalue Problems?

$$y'' + \lambda y = 0, \quad y(0) = 1 \ (\text{BC 1}), \quad y(\pi) = 0 \ (\text{BC 2})$$

70% correctness
30% completeness
neatness

By Eig. Prob.

$\Rightarrow y(x) = c_1 \cos\sqrt{\lambda}x + c_2 \sin\sqrt{\lambda}x$

$y(0) = 1 = c_1$

$\Rightarrow y(\pi) = 0 = 1 \cdot \cos\sqrt{\lambda}\pi + c_2 \sin\sqrt{\lambda}\pi$

$\Rightarrow c_2 = -\dfrac{\cos\sqrt{\lambda}\pi}{\sin\sqrt{\lambda}\pi} = -\cot\sqrt{\lambda}\pi$

$\Rightarrow y(x) = \cos\sqrt{\lambda}x - \cot\sqrt{\lambda}\pi \sin\sqrt{\lambda}\pi$

$\Rightarrow$ non-homogeneous problem with unique soln.

Ex2) $y'' + y = 0 \quad y(0) = 1, \ y(\pi) = a \quad a \in \mathbb{R}$

$\Rightarrow y(x) = c_1 \cos x + c_2 \sin x$

$c_1 = 1, \ -c_1 = a \Rightarrow c_1 = -a$

If $a = -1 \Rightarrow$ regardless of c_2, infinitely many solns exist

i.e. $y = \cos x + c_2 \sin x$

$\Rightarrow$ Non homog. prob with ∞ many or no solns